Iterative Solution of MTL
Based on the Spatial Decomposition and the 2nd order FDTD

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To decrease the computational cost when solving multi-conductor transmission lines (MTL), an iterative solution with spatial decomposition and based on the 2nd order finite-difference time domain (FDTD) is presented. The coupling effects among MTL can be represented by distributed voltage and current sources, by which the MTL can be decoupled. Then the voltages and currents along the lines are evaluated by the 2nd order FDTD and used to updated the equivalent distributed sources. At last, the solution of MTL is obtained using the iterative technique. The proposed method is validated by the numerical examples. And the major advantage of the proposed approach is the inherent parallelism, which will decrease the CPU cost, especially for the MTL system with nonlinear devices.

***Index Terms*—Iterative method, MTL, spatial decomposition, second order FDTD.**

I. Introduction

Basically speaking, the MTL can be solved in the frequency domain and time domain. In the frequency domain, similarity transformation is always used to decouple the partial differential equations describing the MTL [1]. And the results of mode variables need to be transformed back to the actual variables. In time domain, MTL can be solved by FDTD with matrix notation [1]. But for the case of MTL with nonlinear loads, the time-domain method can only be chosen.

The fast growing computational cost is always a serious limiting factor for solving MTL, especially for the nonlinear terminations. In recent years, the transverse partitioning waveform relaxation has been used to evaluate the crosstalk of MTL in frequency domain [2]. It also has been introduced to solve the coupled interconnects in high-speed circuits [3]. In this paper, an iterative method based on the decoupling method and the 2nd order FDTD (DM-FDTD) is presented.

II. Development of the Proposed Approach

A. Iterative solution of the MTL

The MTL system can be described as

$$\frac{\partial \mathbf{V}(x,t)}{\partial x}+\mathbf{L}\frac{\partial \mathbf{I}(x,t)}{\partial t}=\mathbf{E}_x,\ \frac{\partial \mathbf{I}(x,t)}{\partial x}+\mathbf{C}\frac{\partial \mathbf{V}(x,t)}{\partial t}=0, \tag{1}$$

where $\mathbf{V}$ and $\mathbf{I}$ represent the voltage and current vectors. $\mathbf{E}_x$ means the incident electric field along the lines. $\mathbf{L}$, $\mathbf{C}$ are the per-unit-length parameters.

In order to decouple the MTL, $\mathbf{L}$ and $\mathbf{C}$ can be rewritten as

$$\mathbf{L}=\mathbf{L}_{\mathrm{D}}+\mathbf{L}_{\overline{\mathrm{D}}},\ \mathbf{C}=\mathbf{C}_{\mathrm{D}}+\mathbf{C}_{\overline{\mathrm{D}}}, \tag{2}$$

where the subscript D and $\overline{\mathrm{D}}$ means the diagonal and non-diagonal elements, respectively.

Substituting (2) to (1), one can obtain

$$\frac{\partial \mathbf{V}(x,t)}{\partial x}+\mathbf{L}_{\mathrm{D}}\frac{\partial \mathbf{I}(x,t)}{\partial t}=\mathbf{E}_x+\mathbf{e},\ \frac{\partial \mathbf{I}(x,t)}{\partial x}+\mathbf{C}_{\mathrm{D}}\frac{\partial \mathbf{V}(x,t)}{\partial t}=\mathbf{q}, \tag{3}$$

where

$$\mathbf{e}=-\mathbf{L}_{\overline{\mathrm{D}}}\frac{\partial \mathbf{I}(x,t)}{\partial t},\ \mathbf{q}=-\mathbf{C}_{\overline{\mathrm{D}}}\frac{\partial \mathbf{V}(x,t)}{\partial t}. \tag{4}$$

(3) shows that the coupling effects can be represented by the equivalent sources and MTL has been decoupled.

Applying iterative technique to (3), one can obtain a recursive set of decoupled differential equations

$$\frac{\partial \mathbf{V}^{(r+1)}}{\partial x}+\mathbf{L}_{\mathrm{D}}\frac{\partial \mathbf{I}^{(r+1)}}{\partial t}=\mathbf{E}_x+\mathbf{e}^{(r)},\ \frac{\partial \mathbf{I}^{(r+1)}}{\partial x}+\mathbf{C}_{\mathrm{D}}\frac{\partial \mathbf{V}^{(r+1)}}{\partial t}=\mathbf{q}^{(r)}, \tag{5}$$

where r represents the r^{th} iteration step. Fig.1 shows the equivalent circuit of the 1st line at the $(r+1)^{\mathrm{th}}$ iterative step. In Fig.1, x_m (m = 0,1…K) mean the position on the 1st line.

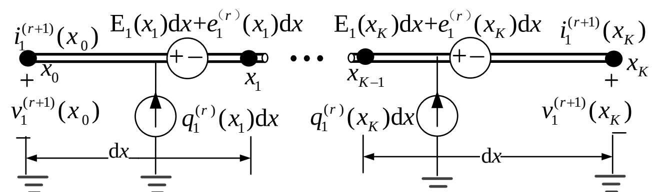

Fig.1. Equivalent circuit of the 1st line at the $(r+1)^{\mathrm{th}}$ iterative step

Based on the above proposition, one can summarize the iterative steps for solving MTL as follows:

Step 1: $\mathbf{e}^{(r)}=0$ and $\mathbf{q}^{(r)}=0$ are assumed with $r=0$.

Step 2: Solving (5) by 2nd order FDTD, one can obtain the solution of $\mathbf{V}^{(r+1)}$ and $\mathbf{I}^{(r+1)}$.

Step 3: Updating $\mathbf{e}^{(r+1)}$ and $\mathbf{q}^{(r+1)}$ using (4), one will repeat step 2 and step 3 until convergence is achieved.

The iterative process will be terminated till the relative difference between two steps is less than the pre-defined tolerance.

B. Solving the decoupled MTL by the 2nd order FDTD

Applying the Taylor's series on the voltage and current in (5), one can obtain the 2nd order FDTD recursive relations when the terms after 2nd order are truncated [4],

$$\begin{aligned} v_k^{n+1}&=v_k^n+\frac{\Delta t}{C}\left(q_k^n-\frac{i_{k+1}^n-i_{k-1}^n}{2\Delta x}\right)+\frac{\Delta t^2}{2LC}\\ &\quad\left(\frac{q_k^{n+1}-q_k^{n-1}}{2\Delta t}-\frac{e_{k+1}^n-e_{k-1}^n}{2\Delta x}+\frac{v_{k+1}^n+v_{k-1}^n-2v_k^n}{\Delta x^2}\right),\\ i_k^{n+1}&=i_k^n+\frac{\Delta t}{L}\left(e_k^n-\frac{v_{k+1}^n-v_{k-1}^n}{2\Delta x}\right)+\frac{\Delta t^2}{2LC}\\ &\quad\left(\frac{e_k^{n+1}-e_k^{n-1}}{2\Delta t}-\frac{q_{k+1}^n-q_{k-1}^n}{2\Delta x}+\frac{i_{k+1}^n+i_{k-1}^n-2i_k^n}{\Delta x^2}\right),\end{aligned} \tag{6}$$

where v, i, q, e, L, C is the element of **V**, **I**, **q**, **e**, **L**, **C** in (5), respectively. Δt and Δx is the time and spatial discretization step, respectively. n means the time step and k means the spatial step.

Actually, the decoupled MTL can be solved correctly by the 1st order FDFD or the 2nd order FDTD. However, the iterative process need the 1st order time derivative of the solutions obtained in the last iterative step, just as shown in (4). For this case, 2nd order FDTD can make iterative process work finely while 1st order FDTD fails to do that, which can be seen from the results of numerical examples.

III. Numerical Examples

A. Ribbon cables

The terminal configurations of a three-wire ribbon cable and the voltage waveform are shown in Fig.2 [1].

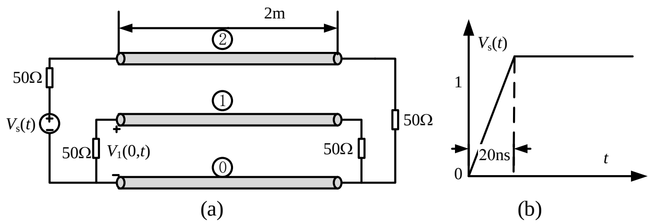

Fig.2 Ribbon cable (a) Terminal configurations (b) Voltage waveform

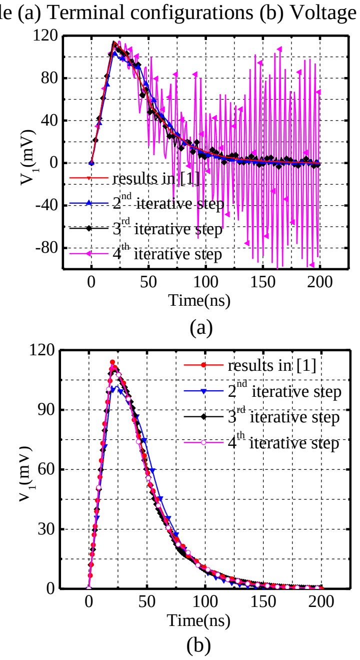

Fig.3. Comparison between the results of proposed method and that in [1]. (a) DM-FDTD with 1st order method. (b) DM-FDTD with 2nd order method.

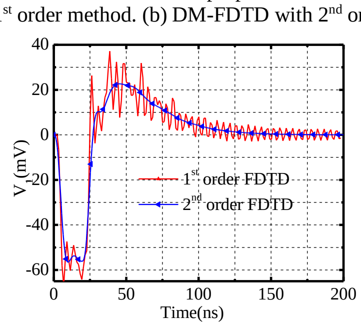

Fig.4. Equivalent voltage sources in the middle of the 2nd line at the 3rd iterative step

Fig.3(a) shows that the results of DM-FDTD with 1st order method will be oscillating at the 3rd iterative step, and will be diverging at the 4th iterative step. This is because the equivalent sources obtained by 1st order FDTD is oscillating, just as shown in Fig.4. And this phenomenon will be aggravated with the iteration. Fig.3(b) shows that the results of the proposed approach can agree well with the results in [1].

B. Busbar of the substation

The terminal configurations of the busbar in the substation and the voltage waveform are shown in Fig.5 [1].

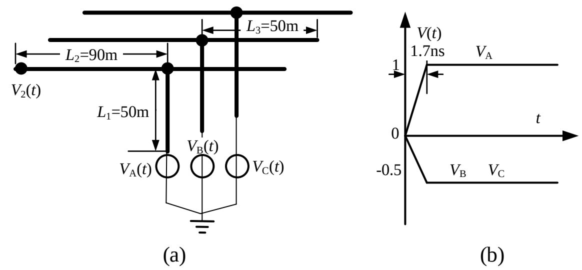

Fig.5 The simplified model of the busbar in the substation (a) Terminal configurations (b) Voltage waveform

Fig.6 shows that the results of the proposed method can almost agree with that in [5]. It also can be seen that the results of the proposed method are smoother.

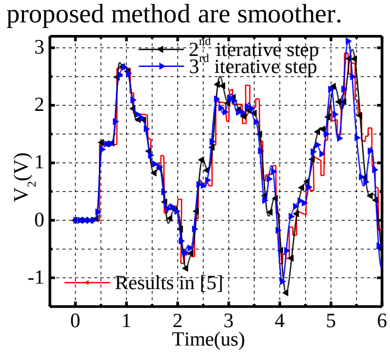

Fig.6. Comparison between the results of proposed method and Bergeron

IV. Conclusion

This paper presents an iterative solution to solve the MTL in time domain. The major advantage is the inherent parallelism. And the further work will involve frequency-dependent parameters and nonlinear loads.

Acknowledgement

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References

[1] C. R. Paul, Analysis of Multiconductor Transmission Lines, 2nd ed. New York, USA: Wiley, 2008, pp. 282-290.

[2] Y. Z. Xie, F. G. Canavero, T. Maesri, et al. "Crosstalk Analysis of Multiconductor Transmission Lines Based on Distributed Analytical Representation and Iterative Technique". *IEEE Trans. EMC*, vol. 52, no. 3, pp. 712-727, Aug. 2010.

[3] N. Nakhla, A. Ruehli, M. Nakhla, et al. "Simulation of coupled interconnects using waveform relaxation and transverse partitioning". *IEEE Trans. Adv. Packag*. vol. 29, no. 1, pp. 78–87, Feb. 2006.

[4] M. Paolone, C. A. Nucci, and F. Rachidi, "A new finite difference time domain scheme for the evaluation of lightning induced overvoltages on multiconductor overhead lines", 5th Int. Conf. Power Syst. Transients, Rio de Janeiro, Brazil, 2001.

[5] T. B. Lu, X. Cui. "Transient Analysis of Wave processes for Multi-Conductor Transmission Lines with Branches Using FDTD". USA: ISEC, vol.2, pp. 699-703, 2000.